Stochastic Geometry Days schedule and abstracts

Monday, May 27th

10h	Welcome
10h30-12h	Céline Duval (1/4)
14h-15h30	Mathew Penrose (1/4)
	Coffee Break
16h-17h30	Céline Duval (2/4)

Tuesday, May 28th

9h-10h30	Mathew Penrose (2/4)
	Coffee Break
11h-12h30	Céline Duval (3/4)
14h15-15h45	Mathew Penrose (3/4)
	Coffee Break
16h15-17h45	Céline Duval (4/4)

Wednesday, May 29th

9h-10h30	Mathew Penrose (4/4)
	Coffee Break
11h-11h45	Franco Severo
11h50-12h20	Antonin Jacquet
14h20-15h05	Christophe Biscio
15h10-15h55	Elena Villa
	Coffee Break
16h30-17h	Sayeh Khaniha
17h05-17h50	Arnaud Poinas

Thursday, May 30th

9h-9h45	Nathanaël Enriquez
9h50-10h20	Thibaut Duboux
Coffee	Break
10h50-11h35	Alice Contat
11h40-12h10	Ludovic Morin
14h-14h45	Pierre Chainais
14h50-15h35	Vytaute Pilipauskaite
Coffee	Break
16h10-16h40	Alexandre Makhlouf
16h45-17h30	Xiaochuan Yang
17h35-18h	RT Meeting
19h30	*Conference dinner*

Friday, May 31th in honour of Anne Estrade

9h-9h45	Yimin Xiao
9h50-10h35	Elena di Bernardino
Coffee	Break
11h00-11h45	Jose Leon
11h50-12h35	Pierre Calka
14h00-16h00	*A festive afternoon with Anne Estrade*

NON-PARAMETRIC INTENSITY ESTIMATION OF SPATIAL POINT PROCESSES BY RANDOM FORESTS

CHRISTOPHE BISCIO
University of Aalborg, Denmark

FRÉDÉRIC LAVANCIER
ENSAI, Rennes, France

The intensity of a spatial point process is one of the first quantity of interest to estimate in presence of real-data. When no covariate is observed, non-parametric kernel estimation is routinely used, but comes with some drawbacks: it adapts poorly to non-convex domain, and the estimation is not consistent in an increasing domain asymptotic regime. When the intensity depends on observed covariates, most estimation methods are parametric. Non-parametric kernel estimation has been extended to this situation, but it appears to be efficient only for a few numbers of covariates, and provides no indication on the importance of each covariate, which is crucial for interpretation.

In this talk, we show how to adapt random forest regression to circumvent these drawbacks and estimate non-parametrically the intensity of a spatial point process with or without covariates, while measuring the importance of each variable in the latter case. Our approach allows to handle non-convex domain together with a large number of covariates. From a theoretical side, we prove that in the case of purely random forests, our method is consistent in both infill and increasing domain asymptotic regime, and may achieve a minimax rate of convergence.

FRACTAL ANNE

PIERRE CALKA
Université de Rouen Normandie

YANN DEMICHEL
Université Paris Nanterre

Abstract. Throughout her career, Anne Estrade has developed and used tools from fractal geometry for studying different kinds of random sets with notable applications to medicine. This talk is intended as a tribute to her and to her body of work in this particular field. At the heart of the presentation is the mysterious notion of Fractal Anne that we unveil and study. The material is based on a joint work with Yann Demichel.

THE STORY OF HOW ANNE TOOK ME ON A EXCURSION SET: SOME THEORETICAL AND COMPUTATIONAL DEVELOPMENTS.

L'histoire de comment Anne m'a emmené dans un ensemble d'excursion : quelques développements théoriques et computationnels.

ELENA DI BERNARDINO

Laboratoire J.A. Dieudonné, UMR CNRS 7351, Université Côte d'Azur, Parc Valrose, 06108 Nice, Cedex 2, France

joint works with: M. Abaach, H. Biermé, R. Cotsakis, C. Duval, A. Estrade, J. R. Léon, T. Opitz

Abstract. The excursion set of a smooth random field carries relevant information in its various geometric measures. Geometric properties of these exceedance regions above a given level provide meaningful theoretical and statistical characterizations for random fields defined on Euclidean domains. Many theoretical results have been obtained for excursions of Gaussian processes and include expected values of the so-called Lipschitz–Killing curvatures (LKCs), such as the area, perimeter and Euler characteristic in two-dimensional Euclidean space. In this talk we will describe a recent series of theoretical and computational contributions in this field. Our aim is to provide answers to questions like:

- How the geometric measures of an excursion set can be inferred from a discrete sample of the excursion set;
- How these measures can be related back to the distributional properties of the random field from which the excursion set was obtained;
- How the excursion set geometry can be used to infer the extremal behavior of random fields

EXCURSION SETS: FROM GEOMETRY TO ESTIMATION

CÉLINE DUVAL
University of Lille, Laboratoire Paul Painlevé, France

Abstract. In this mini-course we indent to show how the geometry of excursion sets of a random field, successfully exploited in many applications, can be used to built statistical procedures. For random field $X : \mathbb{R}^d \mapsto \mathbb{R}$ and a level $u \in \mathbb{R}$, the excursion set of X above u is $\{t \in \mathbb{R}^d, X(t) \geq u\} \cap T$, where $T \subset \mathbb{R}^d$ is the observation window. In dimension $d = 2$, it is a black and white image which indicates whether the field is above or below u (center and right Figures).

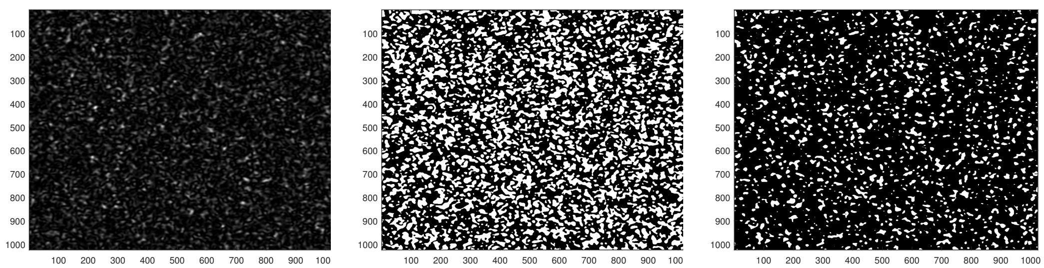

Fig. 0.1 χ^2 field with 2 degrees of freedom observed on a T domain of size $2^{10} \times 2^{10}$ pixels. A realisation of the field (left) and two sets of excursions at $u = 0$ (centre) and $u = 1$ (right).

The geometrical tools considered to study these excursion sets are the Lipschitz Killing curvatures (also called Minkowski functionals or intrinsic volumes). In dimension 2, they correspond to the area, the perimeter and the Euler characteristic, each offering a different point of view on the set. The aim of this mini-course is to introduce these functionals and to show how they can be related to specific parameters or properties of the field and allowing to build statistical procedures (estimation and/or testing). We will discuss some main challenges (e.g. the difficulty of establishing theoretical guarantees or numerical discretization issues) and show they can be addressed.

POISSON-VORONOI MOSAIC ON THE HYPERBOLIC SPACE: WHAT REMAINS WHEN THE POISSON HAS DISAPPEARED?

Nathanaël ENRIQUEZ
Université Paris-Saclay, France

Abstract. I'll try to convince you that, unlike the Euclidean case, the Poisson-Voronoi mosaic on the Hyperbolic space does not converge to the vacuum when the density of the Poisson process tends to 0. I will describe the law of this limit and some of its properties. (Joint work with M. d'Achille, N. Curien, R. Lyons and M. Ünel)

PATTERNS CROSSED BY GEODESICS IN FIRST-PASSAGE PERCOLATION.

ANTONIN JACQUET
Université de Tours – Institut Denis Poisson, France

In first-passage percolation, we consider a family of nonnegative, independent and identically distributed random variables indexed by the set of edges of the graph $\mathbb{Z}^d$, called passage times. The time of a finite path is defined as the sum of the passage times of each of its edges. Geodesics are then the paths with minimal time. A pattern is a local property of the time environment. We fix a pattern and are interested in the number of translates of this pattern crossed by geodesics. The aim is to present a result guaranteeing, under mild conditions, that apart from an event with exponentially small probability, for any geodesic, this number is linear in the distance between the endpoints of the geodesics.

COVERAGE AND CONNECTIVITY OF SPATIAL POINT PROCESSES

MATHEW PENROSE
University of Bath, UK

Abstract.

Suppose A is a bounded domain with smooth boundary in $\mathbf{R}^d$, where $d \geq 2$, and n points are placed uniformly at random in A. Let $Z(n,r)$ be the union of Euclidean balls of radius r centred on these points (a Boolean model with parameters $n \in \mathbf{N}$ and $r > 0$).

The *coverage threshold* T_n is the smallest r such that $A \subset Z(n,r)$, while the *connectivity threshold* K_n is twice the smallest r such that $Z(n,r)$ is a connected set. The random variables K_n and T_n are of interest, for example, in wireless communications and topological data analysis.

In this mini-course we shall describe how to determine the limiting distributions of K_n and T_n, suitably scaled and centred, in the large-n limit. Along the way, we shall investigate some related quantities, and explore some mathematical techniques, that are of interest in their own right. Some parts of the work we shall describe are published (see below) but others are work in progress with Xiaochuan Yang and Frankie Higgs. A provisional plan for the four talks is:

1. Limiting distribution (for large λ) of T'_λ, defined as the furthest distance from any point of A to a homogeneous Poisson process of intensity λ in the whole of $\mathbf{R}^d$ (note that T_n is the furthest distance from any point of A to a binomial point process of of points *within* A). This result dates back to [1, 3] but we may add some refinements if time permits.

2. Limiting distribution of T_n and its Poissonized version. This work was done in [4].

3. Limiting distribution of the *largest nearest neighbour link* L_n, defined to be the smallest r such that each of the n random points is contained in at least one other ball of $Z(n,r)$, besides its own ball. Some of the work for this talk appears in [2].

4. Relating K_n to L_n to complete the description of asymptotics for K_n.

The recent and ongoing work described in this mini-course was supported by EPSRC Grant EP/T028653/1.

References

1. Hall, P. (1985) Distribution of size, structure and number of vacant regions in a high intensity mosaic. Z. Warsch. Verw. gebiete **70**, 237–261.
2. Higgs, Penrose, M.D. and Yang, X. (2024) Covering one point process with another. arXiv:2401.03832
3. Janson, S. (1986) Random coverings in several dimensions. *Acta Math.* **156**, 83–118.
4. Penrose, M.D. (2023) Random Euclidean coverage from within. *Probab. Th. Related Fields* **185**, 747–814.

FRACTIONAL OPERATORS AND FRACTIONALLY INTEGRATED RANDOM FIELDS ON $\mathbb{Z}^{\nu}$

VYTAUTĖ PILIPAUSKAITĖ
Aalborg University, Denmark

DONATAS SURGAILIS
Vilnius University, Lithuania

We consider fractional integral operators $(I-T)^d$, $d \in (-1,1)$, acting on functions $g : \mathbb{Z}^{\nu} \to \mathbb{R}$, where T is the transition operator of a random walk on $\mathbb{Z}^{\nu}$, $\nu \in \mathbb{N}$. We obtain sufficient and necessary conditions for the existence, invertibility and square summability of kernels $\tau(s;d)$, $s \in \mathbb{Z}^{\nu}$, of $(I-T)^d$. Asymptotic behavior of $\tau(s;d)$ as $|s| \to \infty$ is identified following local limit theorem for random walk. A class of fractionally integrated random fields X on $\mathbb{Z}^{\nu}$ solving the difference equation $(I-T)^d X = \varepsilon$ with white noise on the right-hand side is discussed, and their scaling limits. Several examples including fractional lattice Laplace and heat operators are studied in detail.

ATTRACTIVE COUPLING OF DETERMINANTAL POINT PROCESSES USING NONSYMMETRIC KERNELS

ARNAUD POINAS
University of Poitiers, France

Abstract. Determinantal point processes (DPPs for short) are a common class of point processes that model repulsiveness between points. They are defined through a symmetric function called their kernel. From this kernel, we can deduce most of their properties (correlation functions, likelihood, various summary statistics, method of simulation, etc.). DPPs can also be defined using a nonsymmetric kernel, but this case has been rarely studied in the literature since most of the good properties of DPPs require the symmetry of the kernel. In this talk, we will discuss what kinds of properties are satisfied by these more general DPPs and what kinds of assumptions is needed by their kernel to be well-defined. In particular, compared to DPPs with symetric kernels, general DPPs can model attraction between points, and we will show how to use these DPPs in order to create an attractive coupling of DPPs with symmetrical kernels. This would allow us to model marked point processes with repulsion between points of the same marks and attraction between points of different marks.

LEVEL SET PERCOLATION OF SMOOTH GAUSSIAN FIELDS

FRANCO SEVERO
ETH Zurich, Switzerland

Abstract. Given a centered, stationary, smooth Gaussian field f on $\mathbb{R}^d$, we are interested in the geometry of the excursion sets $\{f \leq \ell\} := \{x \in \mathbb{R}^d : f(x) \leq \ell\}$, for $\ell \in \mathbb{R}$. As ℓ varies, these random sets naturally define a percolation model, whose critical point is denoted by ℓ_c. Important examples of such models are the Bargman-Fock field and the Monochromatic Random Wave, which arise as the local limits of random homogeneous polynomials of high degree and random Laplace eigenfunctions of high frequency, respectively. Most of the available literature is devoted to the case $d = 2$, where a special duality property implies, in particular, that $\ell_c = 0$ for basically any planar field. In higher dimensions on the other hand, the lack of duality makes the study more difficult and relatively fewer results are known. In this talk, we shall discuss some of these results, including the sharpness of phase transition in arbitrary dimensions. The latter serves as a crucial step towards a detailed description of the large scale geometry of level sets in the whole off-critical regime $\ell \neq \ell_c$.

MEAN BOUNDARY AND PERIMETER DENSITY OF INHOMOGENEOUS GERM-GRAIN MODELS

ELENA VILLA
Università degli Studi di Milano, Italy

We consider random germ-grain models in $\mathbb{R}^d$ having integer Hausdorff dimension less than d. The mean density, say λ_{Θ_n}, of a random closed set Θ_n with Hasudorff dimension n, is defined to be the density of the measure $\mathbb{E}[\mathscr{H}^n(\Theta_n \cap \cdot)]$ with respect to $\mathscr{H}^d$, whenever it exists. Crucial problems are the evaluation and the pointwise estimation of λ_{Θ_n} in the inhomogeneous setting.

In the first part of the talk, we give sufficient conditions for the existence of $\lambda_{\Theta_n}(x)$. Whenever its theoretical expression is not computable, suitable rectifiability assumptions on the grains allow to provide a local approximation of λ_{Θ_n}, based on a stochastic version of the so-called *n-dimensional Minkowski content* of Θ_n. Such approximation leads to the definition of an estimator of $\lambda_{\Theta_n}(x)$, whose good statistical properties will be also discussed.

In the second part of the talk we will focus on the boundary of full dimensional germ-grain models. Actually, we shall distinguish between the *topological* boundary and the *essential* boundary. The $\mathscr{H}^{d-1}$-measure of the essential boundary coincides with the so-called *perimeter* of the involved set in geometric measure theory. Since the perimeter of a set is invariant under modification of a set of Lebesgue measure zero, the difference between the measure of the topological boundary and the perimeter might be of interest in detecting negligible parts of the set under consideration. The computation of the mean perimeter density and the mean boundary density can be carried out by means of the notion of mean *covariogram* and of mean *outer Minkowski content*, respectively. We will relate these two notions when the perimeter and the $\mathscr{H}^{d-1}$-measure of the topological boundary coincide.

SOME GEOMETRIC PROPERTIES OF ANISOTROPIC GAUSSIAN RANDOM FIELDS

YIMIN XIAO
Michigan State University, East Lansing, USA

Abstract. This talk is concerned with geometric properties of anisotropic Gaussian random fields. We will present some recent results on the hitting probabilities of Gaussian random fields and their applications in studying the existence and Hausdorff dimensions of intersections. If time permits, we will show that, by employing the properties of strong local nonderterminism, one can prove precise results on the local times and level sets of multivariate anisotropic Gaussian random fields.

BOUNDARY EFFECTS IN SOME STOCHASTIC GEOMETRIC MODELS

XIAOCHUAN YANG
Brunel University London, UK

Abstract. Coverage and connectivity of Boolean models are classical problems in stochastic geometry. In this talk, I will present some interesting findings concerning connectivity threshold and coverage threshold in the large n limit, where n is the number of balls in the Boolean model. In most cases, boundary effects play a dominant role in deriving limit theorems for these thresholds. Based on joint works with Frankie Higgs and Mathew Penrose.

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